Fiber bend loss

Example 3.4

Two step index fibers exhibit the following parameters:

(a) a multimode fiber with a core refractive index of 1.500, a relative refractive index difference of 3% and an operating wavelength of 0.82 μm;

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(b) an 8 μm core diameter single-mode fiber with a core refractive index the same as (a), a relative refractive index difference of 0.3% and an operating wavelength of 1.55 μm.

Estimate the critical radius of curvature at which large bending losses occur in both cases.

Solution: (a) The relative refractive index difference is given by Eq. (2.9) as:

$$\Delta = \frac{n_1^2 - n_2^2}{2n_1^2}$$

$\Rightarrow \Delta 2 n_1^2 = n_1^2 - n_2^2$

$\Rightarrow n_2^2 = n_1^2 - \Delta 2 n_1^2$

$= (1.5)^2 - \times \cancel{2.25}(1.5)^2$

n_1 = Core refractive index

relative refractive index = 3% = .03

$\therefore \Delta 2 = .03 \times 2 = .06$

Hence:

$$n_2^2 = n_1^2 - 2\Delta n_1^2 = 2.250 - 0.06 \times 2.250$$
$$= 2.115$$

Using Eq. (3.8) for the multimode fiber critical radius of curvature:

$$R_c \simeq \frac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{3/2}} = \frac{3 \times 2.250 \times 0.82 \times 10^{-6}}{4\pi \times (0.135)^{\frac{3}{2}}}$$
$$= 9\ \mu\text{m}$$
$$= 8.8 \times 10^{-6}$$

λ = operating wavelength

$n_1^2 = 2.25$

$n_2^2 = 2.115$

$.135$

(b) Again, from Eq. (2.9):

$$n_2^2 = n_1^2 - 2\Delta n_1^2 = 2.250 - (0.006 \times 2.250) = 2.237$$

Low relative refractive index = ·003

The cutoff wavelength for the single-mode fiber is given by Eq. (2.98) as:

Single mode operation only occurs above a theoretical cutoff wavelength λ_c.

$$\lambda_c = \frac{2\pi a n_1 (2\Delta)^{\frac{1}{2}}}{2.405}$$

$$= \frac{2\pi \times 4 \times 10^{-6} \times 1.500\,(0.06)^{\frac{1}{2}}}{2.405}$$

$$= 1.214\ \mu m$$

·006

For step index fiber $V_c = 2{\cdot}405$

$$\lambda_c = \frac{2\pi a n_1}{V_c}(2\Delta)^{1/2}$$

V_c = Cutoff normalized frequency

a = maximum core radius $= 8 \div 2 = 4\ \mu m = 4 \times 10^{-6}$

Substituting into Eq. (3.9) for the critical radius of curvature for the single-mode fiber gives:

$$R_{cs} \simeq \frac{20 \times 1.55 \times 10^{-6}}{(0.043)^{\frac{3}{2}}}\left(2.748 - \frac{0.996 \times 1.55 \times 10^{-6}}{1.214 \times 10^{-6}}\right)^{-3} \simeq$$

$$= 34\ mm$$

$$\frac{2\pi \times 4 \times 1{\cdot}5 \times \sqrt{{\cdot}06}}{2{\cdot}405 \times 10^{6}} = \frac{37{\cdot}69 \times \sqrt{{\cdot}06}}{} = {\cdot}24$$

$\sqrt{{\cdot}06} = {\cdot}24$

Example 3.4 shows that the critical radius of curvature for guided modes can be made extremely small (e.g. 9 μm), although this may be in conflict with the preferred design and operational characteristics. Nevertheless, for most practical purposes, the critical radius of curvature is relatively small (even when considering the case of a long-wavelength single-mode fiber, it was found to be around 34 mm) to avoid severe attenuation of the guided mode(s) at fiber bends. However, modes propagating close to cutoff, which are no longer fully guided within the fiber core, may radiate at substantially larger radii of curvature. Thus it is essential that sharp bends, with a radius of curvature approaching the critical radius, are avoided when optical fiber cables are installed. Finally, it is important that microscopic bends with radii of curvature approximating to the fiber radius are not produced in the fiber cabling process. These so-called microbends, which can cause significant losses from cabled fiber, are discussed further in Section 4.7.1.

$$R_c \simeq \frac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{\frac{3}{2}}}$$

- This equation is commonly used in optical fiber communication to calculate the critical bending radius of an optical fiber.

Explanation of Each Term

R_c — Critical Radius of Curvature

- Minimum radius to which an optical fiber can be bent **without significant signal loss**
- If fiber is bent with radius **smaller than** R_c, radiation (bending) loss increases sharply
- Unit: **meters (m)**

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n_1 — Refractive Index of the Core

- Refractive index of the **fiber core**
- Determines how strongly light is confined inside the core
- Dimensionless quantity

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n_2 — Refractive Index of the Cladding

- Refractive index of the **fiber cladding**
- Slightly smaller than n_1
- Dimensionless quantity

$$R_c \simeq \frac{3n_1^2\lambda}{4\pi(n_1^2 - n_2^2)^{\frac{3}{2}}}$$

- This equation is commonly used in optical fiber communication to calculate the critical bending radius of an optical fiber.

λ — Wavelength of Light

- Wavelength of the optical signal propagating in the fiber
- Usually measured in **meters (m)** or **nanometers (nm)**
- Common values: 850 nm, 1310 nm, 1550 nm

π — Mathematical Constant

- Value ≈ **3.1416**
- Arises from circular waveguide geometry

Example 3.2

Silica has an estimated fictive temperature of 1400 K with an isothermal compressibility of 7×10^{-11} m^2 N^{-1} [Ref. 13]. The refractive index and the photoelastic coefficient for silica are 1.46 and 0.286 respectively [Ref. 13]. Determine the theoretical attenuation in decibels per kilometer due to the fundamental Rayleigh scattering in silica at optical wavelengths of 0.63, 1.00 and 1.30 μm. Boltzmann's constant is 1.381×10^{-23} J K^{-1}.

Solution: The Rayleigh scattering coefficient may be obtained from Eq. (3.4) for each wavelength. However, the only variable in each case is the wavelength, and therefore the constant of proportionality of Eq. (3.4) applies in all cases. Hence:

$$\gamma_R = \frac{8\pi^3 n^8 p^2 \beta_c K T_F}{3\lambda^4}$$

$n = 1.46^8 = 20.65$ $\quad p = .286^2 = .082$

γ_R = Rayleigh scattering coefficient

$$= \frac{248.15 \times 20.65 \times 0.082 \times \underbrace{7 \times 10^{-11}}_{\beta_c} \times \underbrace{1.381 \times 10^{-23}}_{K} \times \underbrace{1400}_{T_F}}{3 \times \lambda^4}$$

$$= \frac{1.895 \times 10^{-28}}{\lambda^4} \text{ m}^{-1}$$

At a wavelength of 0.63 μm:

$\lambda^4 = (.63 \times 10^{-6})^4 = 0.158 \times 10^{-24}$

$$\gamma_R = \frac{1.895 \times 10^{-28}}{0.158 \times 10^{-24}} = 1.199 \times 10^{-3} \text{ m}^{-1}$$

The transmission loss factor for 1 kilometer of fiber may be obtained using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-\gamma_R L) = \exp(-1.199 \times 10^{-3} \times 10^3) = 0.301$$

The attenuation due to Rayleigh scattering in decibels per kilometer may be obtained from Eq. (3.1) where:

$$\text{Attenuation} = 10 \log_{10}(1/\mathcal{L}_{km}) = 10 \log_{10} 3.322 = 5.2 \text{ dB km}^{-1}$$

$1/0.301 = 3.322$

At a wavelength of 1.0 μm:

$\lambda^4 = (1 \times 10^{-6})^4 = 10^{-24}$

$$\gamma_R = \frac{1.895 \times 10^{-28}}{10^{-24}} = 1.895 \times 10^{-4} \text{ m}^{-1}$$

Using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-1.895 \times 10^{-4} \times 10^3) = \exp(-0.1895) = 0.827$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.209 = 0.8 \text{ dB km}^{-1}$$

$1/\mathscr{L}_{km} = \frac{1}{0.827} = 1.209$

At a wavelength of 1.30 μm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{2.856 \times 10^{-24}} = 0.664 \times 10^{-4}$$

$\lambda^4 = (1.30 \times 10^{-6})^4 = 2.856 \times 10^{-24}$

Using Eq. (3.5):

$$\mathscr{L}_{km} = \exp(-0.664 \times 10^{-4} \times 10^3) = 0.936$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.069 = 0.3 \text{ dB km}^{-1}$$

$1/0.936 = 1.069$

The theoretical attenuation due to Rayleigh scattering in silica at wavelengths of 0.63, 1.00 and 1.30 μm, from Example 3.2, is 5.2, 0.8 and 0.3 dB km^{-1} respectively. These theoretical results are in reasonable agreement with experimental work. For instance, a low reported value for Rayleigh scattering in silica at a wavelength of 0.6328 μm is 3.9 dB km^{-1} [Ref. 13]. However, values of 4.8 dB km^{-1} [Ref. 14] and 5.4 dB km^{-1} [Ref. 15] have also been reported. The predicted attenuation due to Rayleigh scattering against wavelength is indicated by a dashed line on the attenuation characteristics shown in Figures 3.1 and 3.3.